# The 3D Effect of Bull by Vasarely 

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#### Abstract

In the Vega series of paintings by Vasarely, half spheres bulge out of the plane. We make an attempt to construct transformations that produce this effect. We take a close look at the two spheres in the painting Bull. We visualize parametric stereographic projection and show what parameter choices fit best for the two spheres in the painting. It remains an open issue if Vasarely had the same projection in mind. All the same, the interactive experimental tool can serve as a means to give insight into the mathematical idea behind the painting.


## Bull

During his "Vega" period (1968-1984), Victor Vasarely (1906-1997) produced a series of works with the illusion of a sphere bulging outwards or inwards [1,2]. The common characteristics of these paintings is that a regular grid is deformed to give the 3D effect. Different forms may appear in the squares, and the 3D effect may be enhanced by the coloring of the squares. The paintings inspired mathematicians to create similar artworks [3,4], by using some mathematical models and software to create the images. None of these papers investigated what methods Vasarely may have used to create his paintings. In this paper we take a close look at Bull, a painting from the series on exhibit at the Vasarely Museum in Budapest - see Figure 1.


Figure 1: Bull by V. Vasarely (reproduction from the Vasarely Museum Budapest).
The 3D effect of the Bull is very strong. Casual visitors as well as art historians talk about spheres bulging out of the 2D plane of the canvas. We wanted, in a reverse engineering manner, to reconstruct the idea, and spot the transformation that Vasarely may have had in mind. In this way not only can one understand
the origin of the visual effect in this work, but also the piece of art may be a motivation to explore 3D mappings. In another article, we study of the color patterns of the painting [8].

## Parametric Stereographic Projection

The most straightforward idea is to experiment with the inverse of stereographic mapping. We start from the flattened regular grid, placed in a horizontal plane spanned by $x$ and $y$ coordinate axes. We intersect this plane with a sphere in such a way, that the plane divides the sphere into two identical semi-spheres. For easy reference, we call the top and bottom of the sphere North and South Pole. Then we take the South Pole as projection center, and we project the grid points of the plane (within the circle which the sphere dissected) to the upper half of the sphere. Finally, we perform an orthogonal projection from the upper half of the sphere to the plane, and replace the original grid points by their images, see Figure 2 left.


Figure 2: Stereographic projection using half sphere, where $C=O$ and $Q=S$ (a) and parametrized version, by $C O=s z$ and $Q O=p z$ parameters $(b)$. The grid point $P$ is transformed to $P_{2}$.

The first part of the transformation - projecting the grid points to the sphere - is the inverse of the stereographic mapping [5], which is normally used the other way around, creating a planar representation of a pattern on a sphere (typically, to gain a map of the Earth). The mathematical formulas to compute the coordinates on the sphere are easy to derive, by calculating where the line of projection intersects the upper half of the sphere. It is also evident that the image of a straight line will be a circle on the sphere, going through the South Pole, but not containing the South Pole itself. The orthogonal projection of such a circle is an ellipse. Hence the transformation yields arcs of ellipses.

The radius $r$ and the $x$ and $y$ coordinates of the center of the sphere is defined by the original painting (with 500 pixel sides as reference for the numerical values given in pixels), thus the generation is straightforward. The result is shown in Figure 3. In spite of the "similar effect", Figure 3 does not show a good match with the original painting. Hence, the hypothesis that half-spheres are visible in Bull, must be given up. Of course, this does not imply that Vasarely did not intend to draw half-spheres.

We can experiment with two relaxations of the conditions of the projections:

- the center of projection may move from the South pole up or down along the diagonal of the sphere, indicated by parameter $p z$,
- the center of the sphere may be pushed downwards (according to parameter $s z$ ) so that less than the half remains above the plane and is used as projection surface. One has to assure that the intersection of the sphere and the plane remains a given circle, thus the change of $s z$ implies the change of the radius of the sphere, see Figure 2b.


Figure 3: First reconstruction, based on stereographic projection as explained in Figure $2 a$.
After some experimentation, we managed to get a much better - but still not perfect - match to the painting. Interestingly, for the two spheres in the painting different parameters provide the best match, namely:
for the top sphere part: $\quad r=220, p z=-72, p s=-40, R=223,6 ;$
for the corner sphere part: $\quad r=330, p z=-91, p s=-60, R=335,4$.
As $p s$ and $p z$ are relatively small, we may still say that the shown forms are almost half and quarter spheres, and the center of projection is close to the center of the spheres. The result can be seen in Figure 4. While Figure 4a suggest a good match with the artwork, in Figure 4b we see the deviation from it, by imposing the computer-generated transformation of the planar grid on the original painting by Vasarely.


Figure 4: The reconstructed version (left), and its grid superimposed on Vasarely's Bull (right).

Needless to say, with the mismatch we do not suggest that Vasarely's work is not "perfect". Actually, the small deviations from the strict geometric model make the effect of bulging more dramatic.

## Further Issues

The above "reverse engineering" endeavor was triggered to find out what could be the "program", the algorithm Vasarely used. He did not have access to programming means in his time, but he (or an assistant) could have plotted the transformed grid by calculating the coordinates. After getting experience with producing several paintings with bulging spheres, he probably disregarded such an aid and could draw the transformed lines by hand. It would be intriguing to find written or oral reference of the production process he actually pursued. Based on the following words by Vasarely in an interview [6, p. 69], we suspect that he (or his assistant) created the transformed grids free hand: "The drawing became free, at least in the deformed parts, but still not excluding the possibility of the collective execution."

The above reconstructions were produced by a piece of software written in Processing [7], for the purpose. The software allows investigation of all other Vega paintings of similar principles, including where circles are placed inside the squares, and with other patterns than square grids. It would be also interesting to investigate whether in other Vega paintings, similar parameter sets produce a good, or even perfect match?

Finally, we strongly believe that such hands-on investigations help the observers to understand the principle behind the painting, the craftsmanship of the maker, and also the underlying math. Currently we are working on embedding the above outlined investigations into an educational application for visitors, to be available on-line [9], in cooperation with the Museum of Fine Arts Vasarely Museum in Budapest.

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